

SAMPLE PAPER 4: PAPER 1

QUESTION 9 (50 MARKS)

Question 9 (a) (i)

$$h = 1 + x$$

Question 9 (a) (ii)

$$x^2 + r^2 = 1^2$$

$$r = \sqrt{1 - x^2}$$

Question 9 (b)

$$\begin{aligned} V &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \pi (1 - x^2)(1 + x) \\ &= \frac{1}{3} \pi (1 + x - x^2 + x^3) \end{aligned}$$

Question 9 (c)

$$V = \frac{1}{3} \pi (1 + x - x^2 + x^3)$$

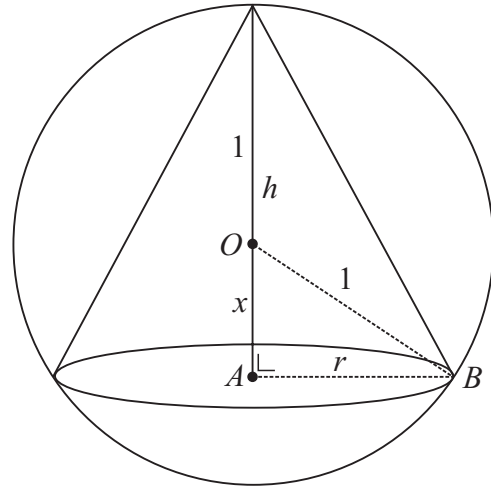
$$\frac{dV}{dx} = \frac{1}{3} \pi (1 - 2x - 3x^2) = 0$$

$$3x^2 + 2x - 1 = 0$$

$$(3x - 1)(x + 1) = 0$$

$$x = \cancel{-1}, \frac{1}{3}$$

$$\therefore V_{\text{Max.}} = \frac{1}{3} \pi \left(1 + \left(\frac{1}{3}\right) - \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^3\right) = \frac{32}{81} \pi$$



Question 9 (d)

$$x = \frac{1}{3}$$

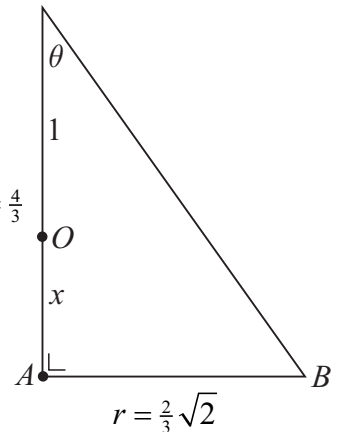
$$h = 1 + x = \frac{4}{3}$$

$$r = \sqrt{1 - x^2} = \sqrt{1 - \left(\frac{1}{3}\right)^2} = \sqrt{\frac{8}{9}} = \frac{2}{3} \sqrt{2}$$

$$\tan \theta = \frac{r}{h} = \frac{\frac{2}{3} \sqrt{2}}{\frac{4}{3}} = \frac{1}{\sqrt{2}}$$

$$\theta = \tan^{-1} \left(\frac{1}{\sqrt{2}} \right) \approx 35^\circ$$

$$h = \frac{4}{3}$$



Question 9 (e)

$$\text{Cone: } \frac{dV}{dt} = -0.2 \text{ cm}^3 \text{ s}^{-1}$$

$$\text{Sphere: } \frac{dV}{dt} = +0.2 \text{ cm}^3 \text{ s}^{-1}$$

$$V = \pi h^2 - \frac{1}{3} \pi h^3$$

$$\frac{dV}{dh} = 2\pi h - \pi h^2$$

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} = (2\pi h - \pi h^2) \frac{dh}{dt}$$

$$\left(\frac{dV}{dt} \right)_{h=0.3} = 0.2 = [2\pi(0.3) - \pi(0.3)^2] \frac{dh}{dt}$$

$$\therefore \frac{dh}{dt} = \frac{0.2}{[2\pi(0.3) - \pi(0.3)^2]} = 0.125 \text{ cm s}^{-1}$$

